ADVANCED GCE UNIT **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

THURSDAY 18 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Fig.1 shows the graphs of $y = |x|$ and $y = |x - 2| + 1$. The point P is the minimum point of $y = |x - 2| + 1$, and Q is the point of intersection of the two graphs.

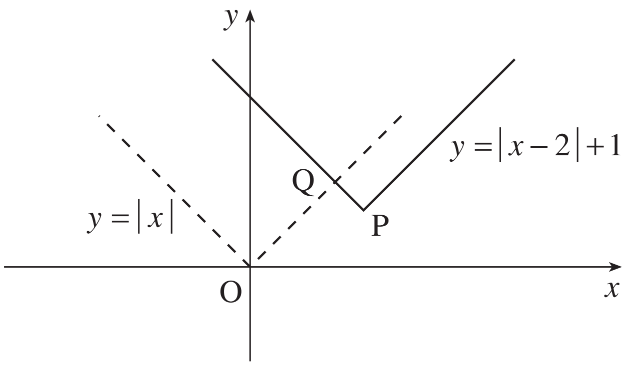

Fig. 1

(i) Write down the coordinates of P. [1]

(ii) Verify that the y-coordinate of Q is $1\frac{1}{2}$. [4]

2 Evaluate $\int_1^2 x^2 \ln x \, dx$, giving your answer in an exact form. [5]

3 The value £V of a car is modelled by the equation $V = Ae^{-kt}$, where t is the age of the car in years and A and k are constants. Its value when new is £10 000, and after 3 years its value is £6000.

(i) Find the values of A and k. [5]

(ii) Find the age of the car when its value is £2000. [2]

4 Use the method of exhaustion to prove the following result.

No 1- or 2-digit perfect square ends in 2, 3, 7 or 8

State a generalisation of this result. [3]

5 The equation of a curve is $y = \dfrac{x^2}{2x + 1}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x(x + 1)}{(2x + 1)^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve. You need not determine their nature. [4]

6 Fig. 6 shows the triangle OAP, where O is the origin and A is the point $(0, 3)$. The point $P(x, 0)$ moves on the positive x-axis. The point $Q(0, y)$ moves between O and A in such a way that $AQ + AP = 6$.

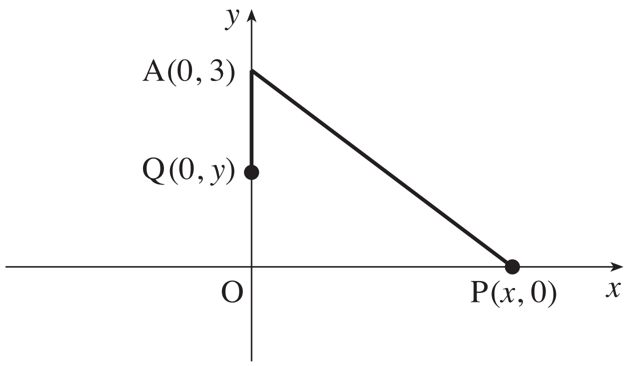

Fig. 6

(i) Write down the length AQ in terms of y. Hence find AP in terms of y, and show that

$$(y+3)^2 = x^2 + 9.$$ [3]

(ii) Use this result to show that $\frac{dy}{dx} = \frac{x}{y+3}$. [2]

(iii) When $x = 4$ and $y = 2$, $\frac{dx}{dt} = 2$. Calculate $\frac{dy}{dt}$ at this time. [3]

Section B (36 marks)

7 Fig. 7 shows part of the curve $y = f(x)$, where $f(x) = x\sqrt{1+x}$. The curve meets the x-axis at the origin and at the point P.

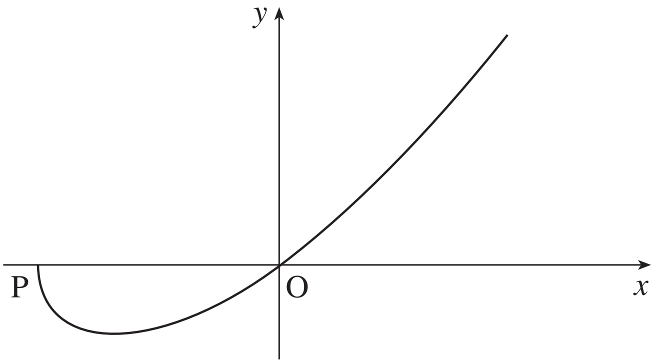

Fig. 7

(i) Verify that the point P has coordinates $(-1, 0)$. Hence state the domain of the function $f(x)$. [2]

(ii) Show that $\dfrac{dy}{dx} = \dfrac{2+3x}{2\sqrt{1+x}}$. [4]

(iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function. [4]

(iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{0}^{1} \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du.$$

Hence find the area of the region enclosed by the curve and the x-axis. [8]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 1)^2 \quad \text{for } x \geqslant 0.$$

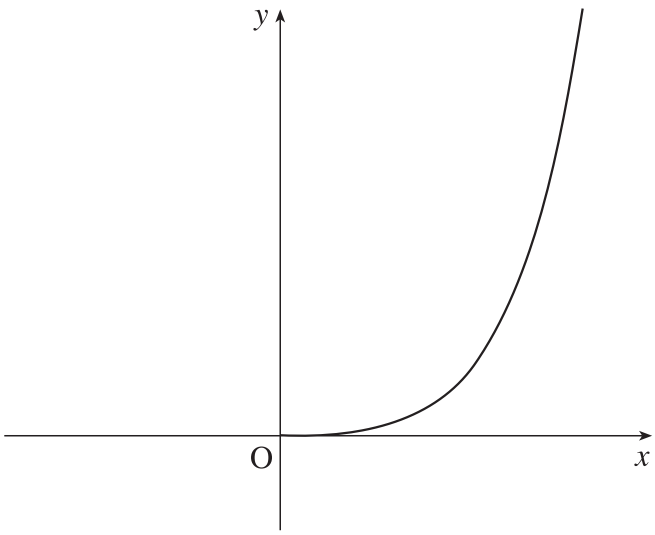

Fig. 8

(i) Find $\mathrm{f}'(x)$, and hence calculate the gradient of the curve $y = \mathrm{f}(x)$ at the origin and at the point $(\ln 2, 1)$. [5]

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \ln(1 + \sqrt{x})$ for $x \geqslant 0$.

(ii) Show that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are inverse functions. Hence sketch the graph of $y = \mathrm{g}(x)$.

Write down the gradient of the curve $y = \mathrm{g}(x)$ at the point $(1, \ln 2)$. [5]

(iii) Show that $\int (\mathrm{e}^x - 1)^2 \, \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^x + x + c$.

Hence evaluate $\int_0^{\ln 2} (\mathrm{e}^x - 1)^2 \, \mathrm{d}x$, giving your answer in an exact form. [5]

(iv) Using your answer to part **(iii)**, calculate the area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the line $x = 1$. [3]

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